Quantitative minimalism and scaling laws

- …from the summary of what I said at 2015 PJP memorial workshop…
- Curvefits, scaling laws and Correlations – same or different?
- Can we succeed in "understanding" and converting it to correlations by simply invoking dimensionless numbers or do we need to do some thinking?
- Pan fire behavior as an example – status of a mixed up past and way forward.
- …from 2016 PJP memorial workshop on g-phase behavior…. and the need for a "clean" experimental design
- Data form colleagues at FCRC, JU – Sowrirajan, Shiva kumar and colleagues
- Thinking and analysis towards a correlation.

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I had stated in 2015 PJP mem. meeting... the summary statement as...

Summing up

- Quantitative minimalism refers to extracting the appropriate elements in a mathematical functional form to create (or help) understanding.
- It forces one to reduce "excesses" – opposite of "taking all things into account"
- Example: for one may compute in all detail – like DNS for instance – may reveal little of primal causes – unless of course, efforts are made to extract the behavior based on hypothesis. An example of very slow progress is understanding turbulent flows.
- Working towards quantitative minimalism will allow deeper, awkward questions to be asked with oneself that will surely bring greater understanding and of course, sobriety – scientific (combustion science here) or whatever one cares to think about!

I think this is so important that I want to present the approach with respect to a specific problem outside the broad aerospace field and show some results.

But, before that.....

Curvefits, Correlations, Scaling laws and – same or different?

- If you have some data of "*y*" vs. "*x*" you do a curve fit even when there are a number of parameters affecting it. It should not get elevated into a "theory" or a law. Closest example is Vielle's law – $\acute{r} = a\, p_c^n$. The parameters vary with composition, etc and it is simply a curve fit – there is no substantive physical content in it.
- Correlations in their better form depend on the fact that the process occurs through physical variables through dimensionless quantities.
- Scaling approach is inherently physically inspired mathematics. There are enormous number of examples and a number of scientists (physicists largely) in engineering and biology.
- I will take for illustration only two examples. The first one is the 1-d premixed flame propagation speed (burning velocity) and the second one is related to pulsating buoyancy dominated diffusion flames.

From Williams' book on Combustion Theory for 1-d premixed flame burning velocity.

conduction from the reaction zone. The rate at which heat is conducted upstream is roughly $\lambda\, dT/dx \approx \lambda(T_\infty - T_0)/\delta$ (energy per unit area per second), where λ is the mean thermal conductivity of the gas, T is the temperature, x is the distance normal to the wave, and the subscripts 0 and ∞ identify conditions upstream and downstream of the wave, respectively. If the wave is adiabatic so that no energy is lost downstream or from the sides of the wave, then energy conservation implies that $q = c_p(T_\infty - T_0)$ (where c_p is an average specific heat of the mixture) and that the entire heat released must be conducted upstream, that is, $qw\delta \approx \lambda(T_\infty - T_0)/\delta$. These results lead to the equation $c_p(T_\infty - T_0)w\delta \approx \lambda(T_\infty - T_0)/\delta$, which implies that the thickness of the wave is

$$\delta \approx \sqrt{\lambda/c_p w}. \qquad (1)$$

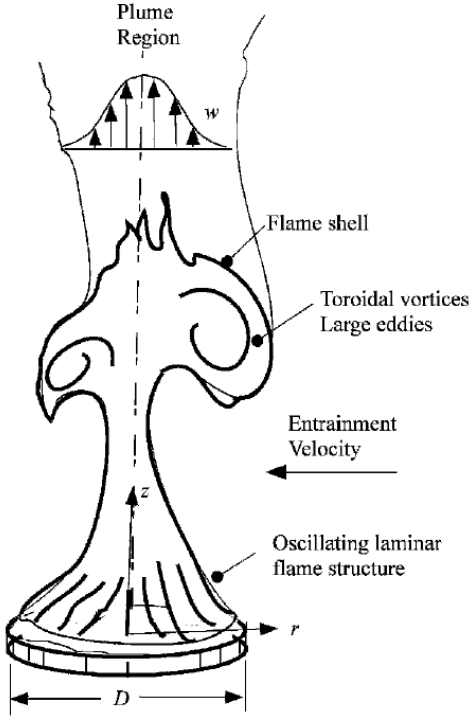
The burning velocity v_0 can be related to this wave thickness as follows. The mass of combustible material per unit area per second flowing into the wave is $\rho_0 v_0$, where ρ_0 is the density of the initial combustible gas mixture. The deflagration wave consumes these reactants at a rate $w\delta$ (mass per unit area per second). Hence mass conservation implies that $\rho_0 v_0 = w\delta$, which, in conjunction with equation (1), yields

$$v_0 \approx (1/\rho_0)\sqrt{(\lambda/c_p)w}. \qquad (2)$$

The scaling approach is visible here.
The final result appears deceptively simple.
In fact, it actually is. All the work done by various researchers in computing the flame structure and its behavior in various conditions cannot violate this including the effects of composition and initial temperature.

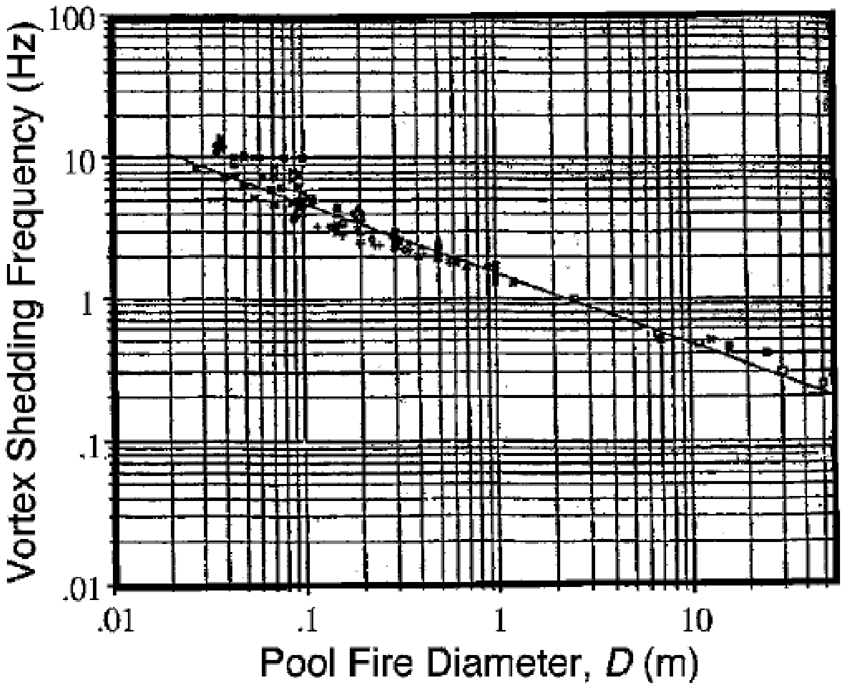
This should not be interpreted that all other research is irrelevant. But when one has complex physics to address, this approach is the most needed one. It can allow you to capture the essence beautifully and simply.

Pan Fire



$$\frac{\dot{m}_e}{\rho_\infty \sqrt{g}\, D^{5/2}} = C_e \left(\frac{z}{D}\right)^{1/2} \left[1 + 2C_\ell \left(\frac{z}{D}\right)\right]^2$$

$\dot{m}_e$ = entrainment mass flow rate



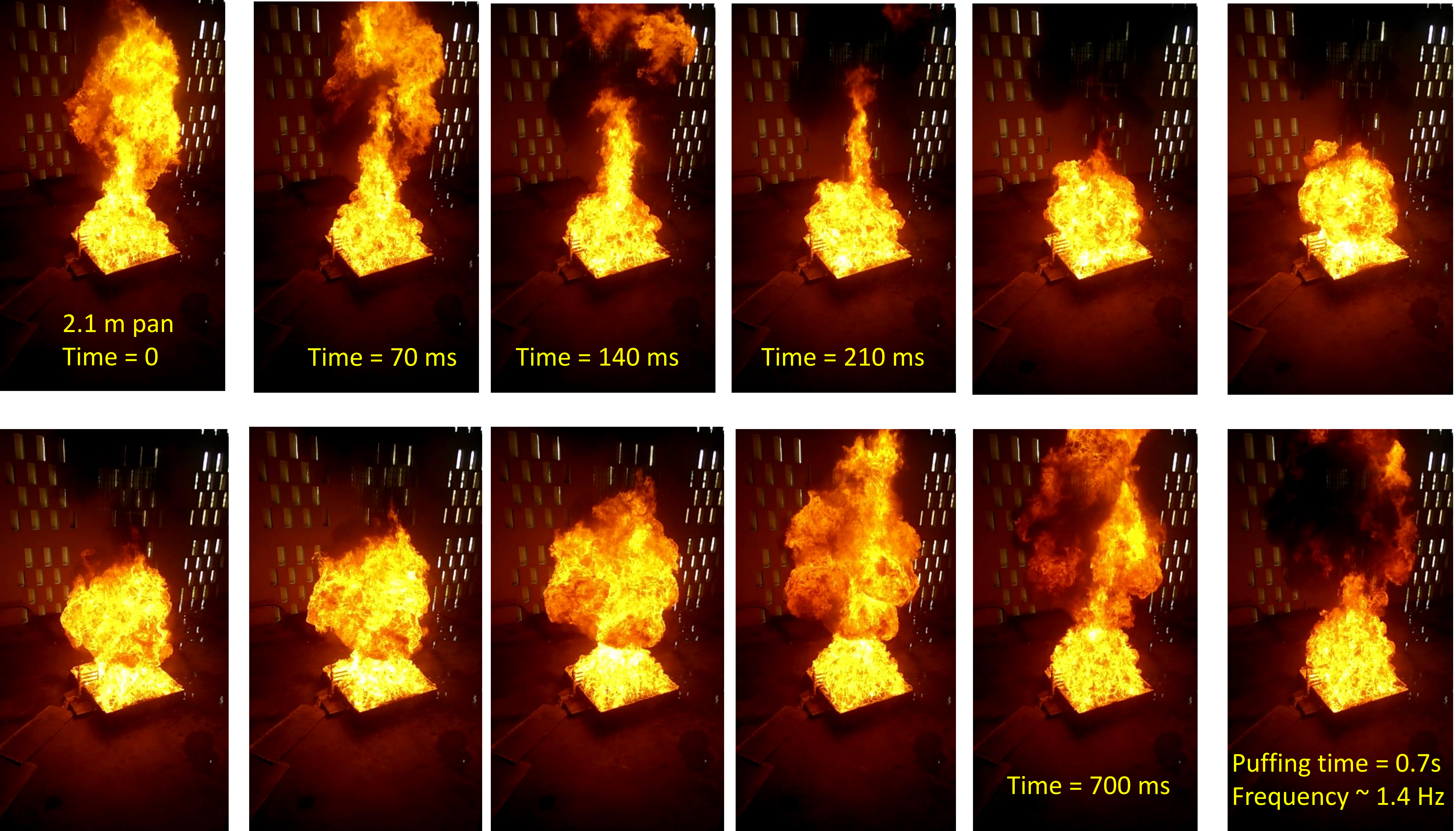
Experiments show f = 0.5 $\sqrt{g/D}$

The simplest argument that can be made is as follows. In a buoyancy dominated flame such as what you see, acceleration due to gravity and the physical dimension are the important physical variables. The time scale that controls this is ~ $\sqrt{D/g}$. So frequency is inverse of this.

Thus,

- Scaling approach is far superior to curve fit. Curve fits can be practiced with dimensional variables (as usually done) and do not carry great value because they have no greater validity outside of the range in which the data for the fit has been used.
- Scaling approach has inbuilt in it dimensionless approach most usually….but there are exceptions.
- In liquid rockets one uses the idea of L^* (as also in solid rocket instability). Typically the combustion chamber of the liquid engine is sized in terms of a value of L^* typically of 0.8 to 1 m. How can this be justified?
- A certain value for L^* is provided to complete the combustion within the stay time inside the rocket.
- The reference times are: drop vaporization times and so lengths required to accomplish vaporization as also high temp. and high pr. chemistry. It is the ratio that really matters.
- Since there is no simple way of characterizing them, for engineering convenience, a dimensional value of L^* is used.

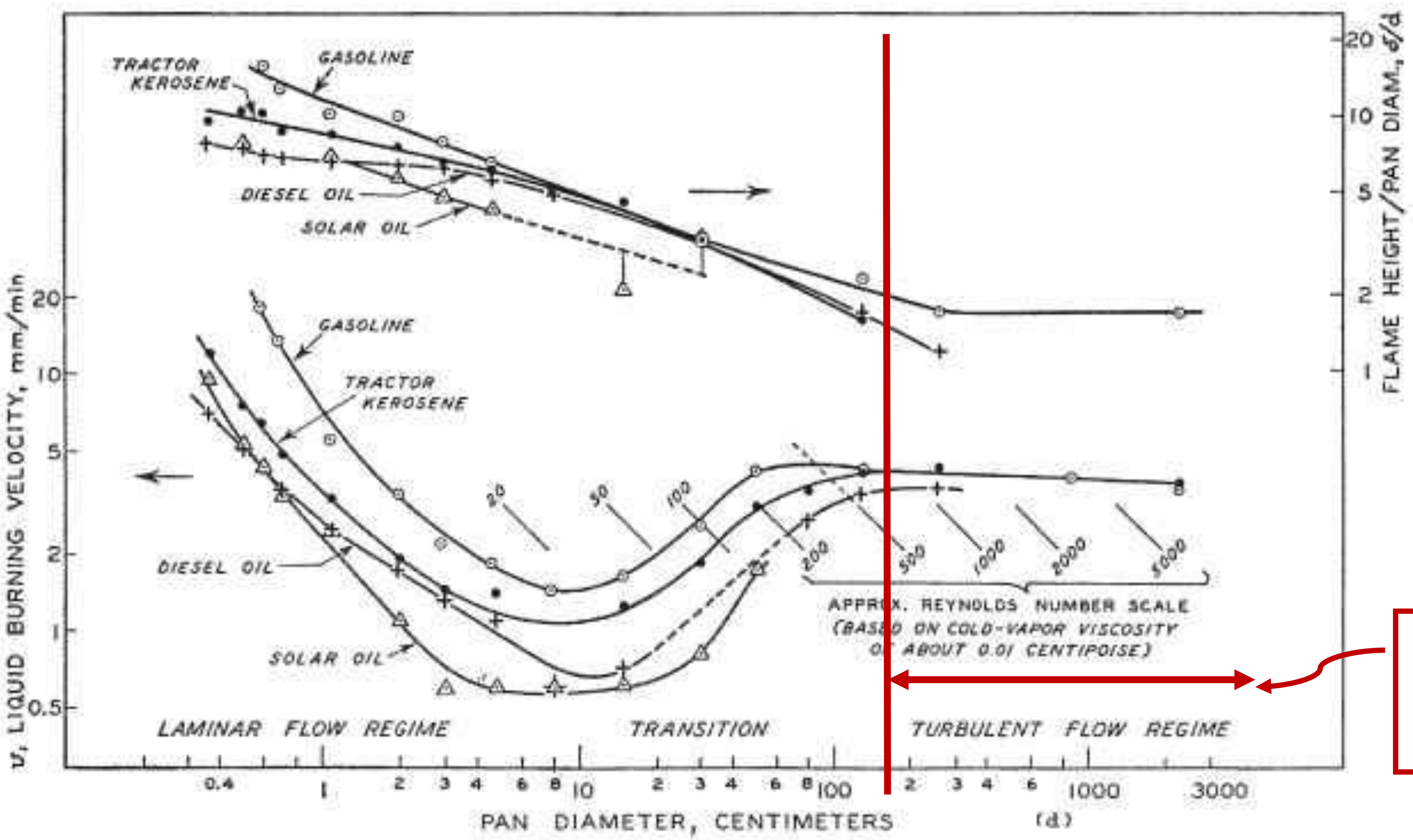
…………………we will use such ideas for understanding and obtaining a correlation for the burn rate of a pan fire…..

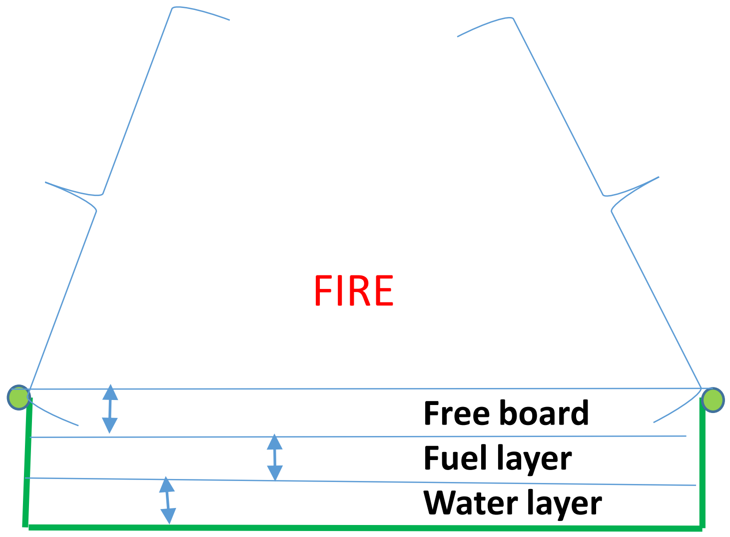

2.1 m pan
Time = 0
Time = 70 ms
Time = 140 ms
Time = 210 ms
Time = 700 ms
Puffing time = 0.7s
Frequency ~ 1.4 Hz

Pan fire behavior as an example – status of a mixed-up past and way forward.

- Pan fires are standard fires used for qualifying fire extinguishing foams and solid powders (DCP – dry chemical powder – of mono-ammonium phosphate).
- Their combustion behavior has been studied and reported by a number of scientists: Babrauskas (Russia), Hottel and his diagram, Koseki, Hamins (NIST, USA), John deRis, Fernandez Pello and several others. Joulain has set out a review in 1998.
- The data from several sources were put together by Hottel (1961) and the diagram of the burn rate measured in terms of mm/min vs pan diameter was presented.
- This seems to have become a "gospel truth" in fire literature, for it is also old…57 years! Like gospel truths, it has many failings.
- The first one is that the burn rate in mm/min as is set out is a parameter that shrouds the "burn rate" which should technically be in terms of mass flux, because the density of fuels varies from 680 kg/m^3 for n-heptane which is a standard fuel in fire qualification tests to 850 kg/m^3 for diesel.
- Peak burn rate for pans of 200 mm and beyond is about 4.8 mm/min [~54 g/m^{2}s for n-heptane, ρ_l = 680 kg/m^3 and 66.4 g/m^{2}s for diesel, ρ_l = 850 kg/m^3] as can be seen…..

Hottel's Correlation and issues





At large pan sizes, only Radiation matters.. Is it true?

But, the burn rate of the fuel in the pans depends, in addition on fuel depth, free board, initial temperature of the fuel, depth of water over which fuel floats, the pan lip geometry,… and so this plot is highly deceptive, a fact to be realized seriously yet!……to prove this point, data from others and ours

Some correlations from literature – but no validation against experiments

From Joulain, 27th symp combustion, 1998, pp 2691 – 2706 - Correlation due to Fernadez Pello

$$\dot{m}'' = 0.15 \frac{\mu_\infty}{PR^2} \left[\frac{g(\rho_\infty - \rho_f)}{v_\infty^2 \rho_\infty} \right]^{1/3} B \left[\frac{\ln(1 + B)}{B} \right]^{1/3} + \chi_R \dot{Q}/DL_v + \dot{q}''_e/L_v - \dot{q}_{rs}/L_v \qquad (10)$$

where D is pool diameter, L_v is heat of vaporization, B is mass transfer number, Pr is Prandtl number, g is gravity, μ_∞ is ambient gas viscosity, ρ_∞ is ambient gas density, ρ_f is density in the flame, χ_R is radiative fraction, $\dot{Q}$ is heat-release rate, and $\dot{q}''_e$, $\dot{q}''_{rs}$ are external and reradiation flux from the surface.

CHAPTER 3

Heat transfer to objects in pool fires

J.P. Spinti, J.N. Thornock, E.G. Eddings, P.J. Smith & A.F. Sarofim
Department of Chemical Engineering, University of Utah, USA.

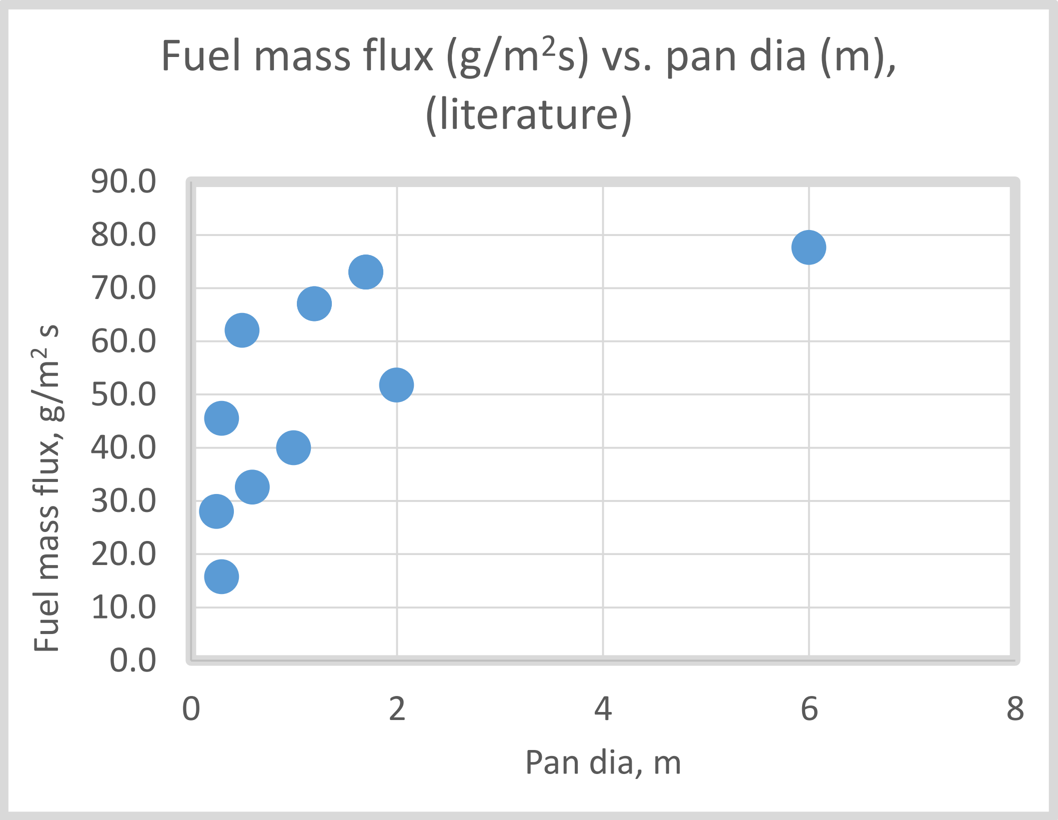
$$\dot{m}'' \Delta h_{vap} = q'' = \frac{4K}{d}(T_F - T_o) + h(T_F - T_o) + \sigma F(T_F^4 - T_o^4)(1 - e^{-Kad})$$

The first term on the right-hand side of eqn (1) represents conduction from the rim of the pan at the flame temperature, T_F, to the liquid at T_o, where K is the liquid conductivity and d is the pan diameter. The second term represents convection from the flame to the liquid, where h is the convective heat transfer coefficient. The third term represents the radiation from the flame, where F is the view factor from the flame to the pan, σ is the Stefan–Boltzmann constant, K is the absorption coefficient in the flame, and a is the ratio of the mean beam length to the pan diameter.

Note that fuel depth is not a parameter In these expressions

Earlier experimental data

Reference	Pan matl	Dia	Mea n ṁ''	Mean ṙ	Fuel - over water?
		m	g/m^2 s	mm/ min	
Kung H C etal (1982)	MS	1.2	67	5.91	?
Kung H C etal (1982)	MS	1.7	73	6.44	?
Tarifa CS (1967)	MS	0.25	28	2.47	?
Tarifa CS (1967)	MS	0.5	62	5.47	?
Hiroshi, koseki etal (1988)	MS	0.3	16	1.39	30 mm, Yes
Hiroshi, koseki etal (1988)	MS	0.6	33	2.87	30 mm, Yes
Hiroshi, koseki etal (1988)	MS	1	40	3.52	30 mm, Yes
Hiroshi, koseki etal (1988)	MS	2	52	4.57	30 mm, Yes
Hiroshi, koseki etal (1988)	MS	6	78	6.85	30 mm, Yes
Kung, Stat..19thsymp, 1982	MS	1.2	67	5.91	
Kung, Stat..19thsymp, 1983	MS	1.7	73	6.44	
Hamins, Kashiwagi, 1995	SS	0.3	46	4.01	



The researchers do not document adequately all the details…..mostly because they must have thought as being unimportant…as are the reviewers of these publications in journals of significance

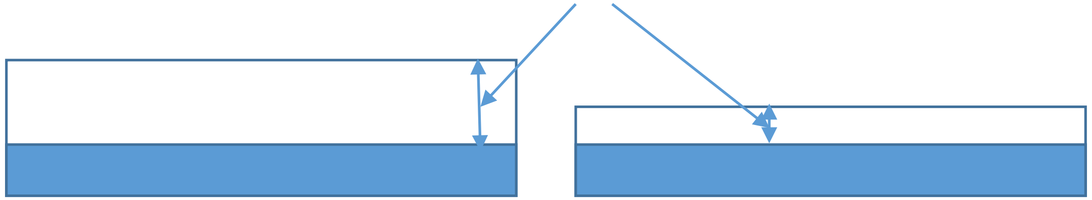
For these reasons, systematic experiments were launched at JU – FCRC with different pans

The pans used for testing at at FCRC and why?

C50060MS3 → Circular pan of 500 mm dia
60 mm deep and 3 mm thick of MS

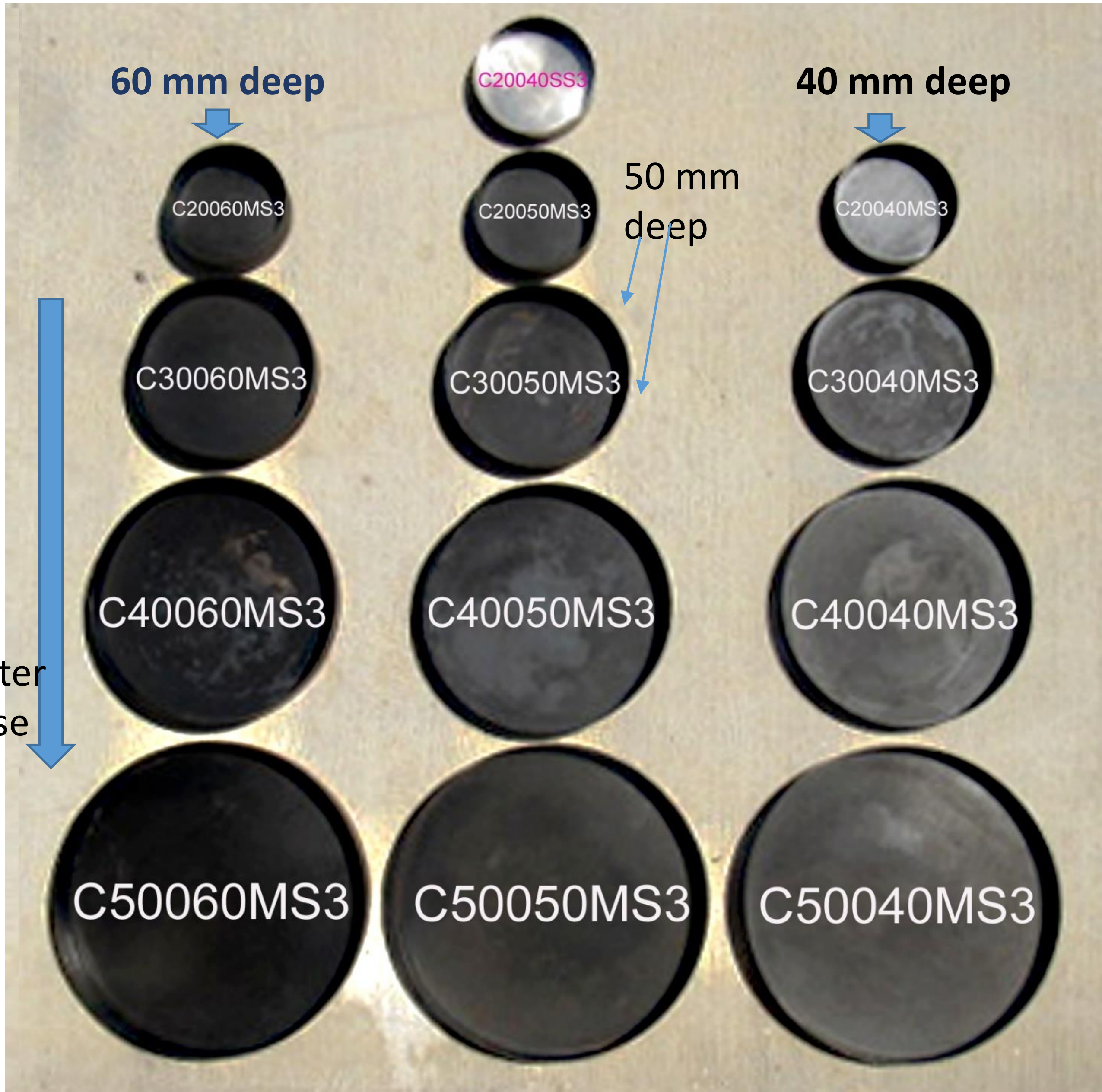
Why so many similar pans? – Diameter is to be varied and we need to control free board and fuel depth independently

Free board can be varied with the same fuel thickness



Diameter increase

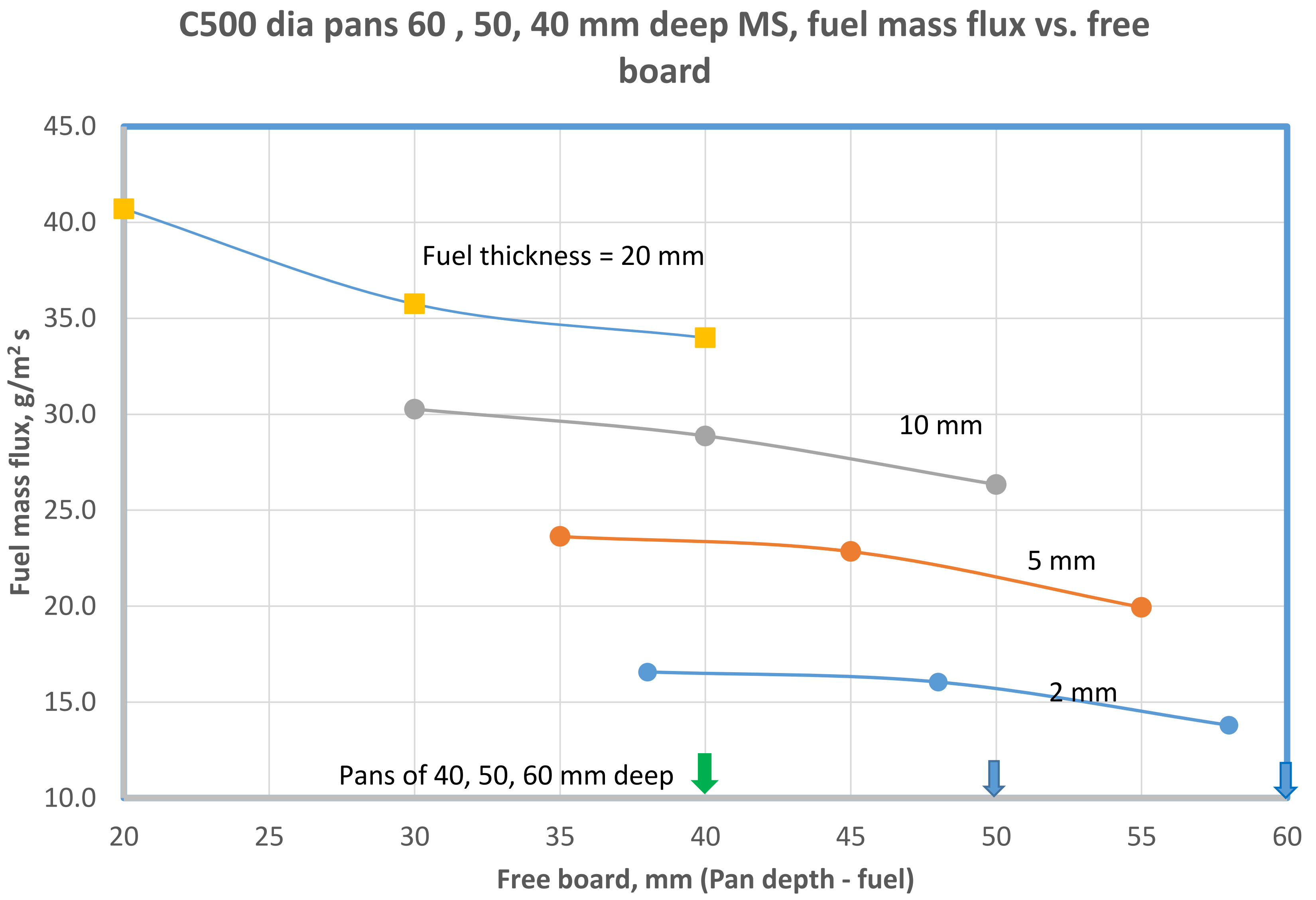
Also a 2000 mm dia pan, 145 mm deep.



Summary of the experiments

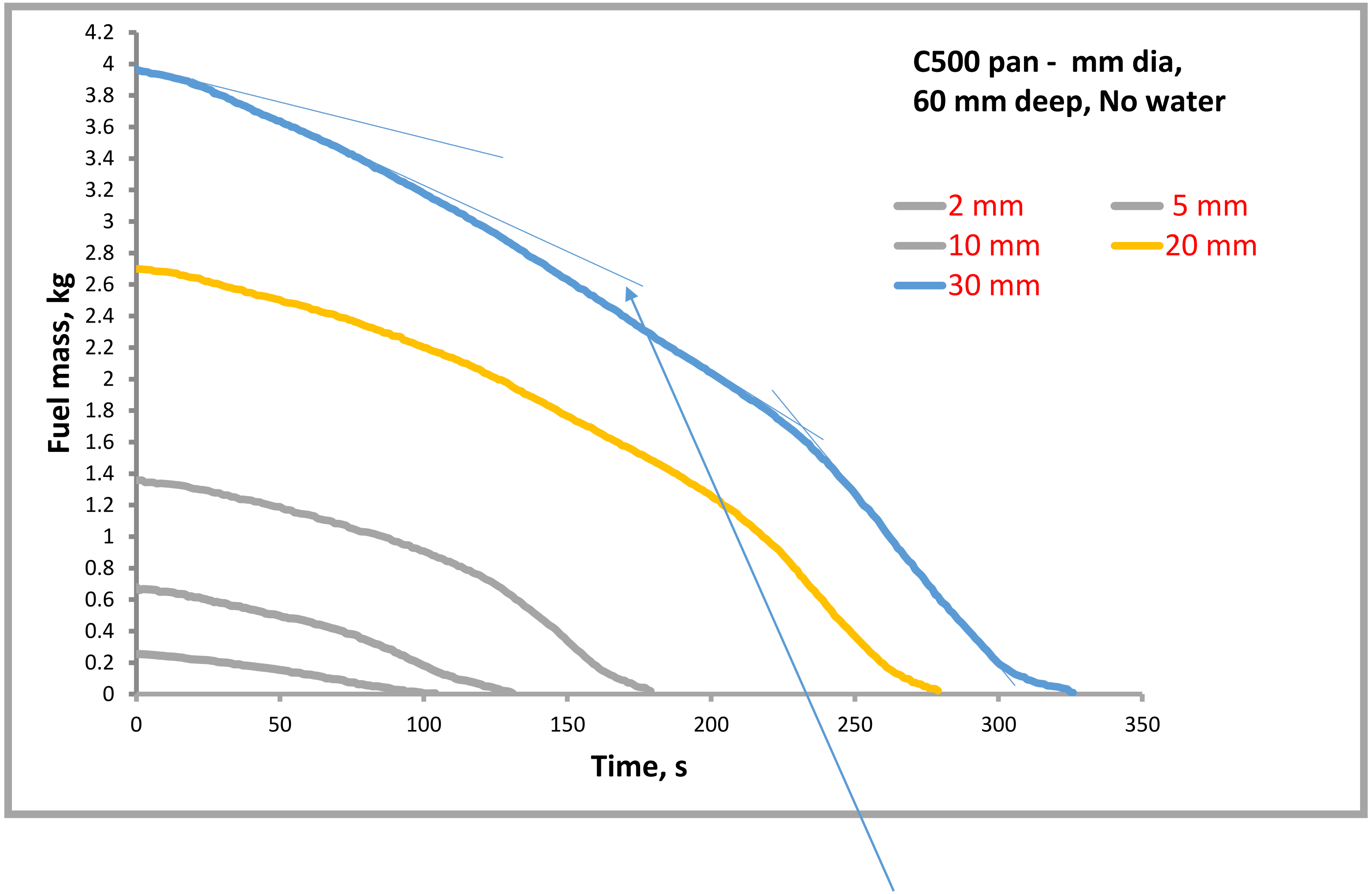
1. All the experiments were conducted by Dr. Sowri rajan and Mr. Shiva kumar at FCRC fire lab (to be described by Prof Dixit later)
2. Specific experiments for 500 mm pan were repeated to check the repeatability on the same day, in a continuous mode by bringing the conditions of the pan to same as at the beginning of the test each tim.
3. The repeatability was good – to within ± 3 %.
4. Yet the experimental data on the same pans done at the same conditions done at different times and different days showed differences of ± 10 %.
5. The precise reasons for this behavior are not fully identified. One suspected serious cause is random wind that can cause additional gas phase flux as also heat transferred to the pan walls.
6. It is possible that this situation may not get improved in practice because, unlike forced convection conditions, free convective conditions can vary.
7. However, significant new results (compared to literature) have emerged…….

Free Board effect



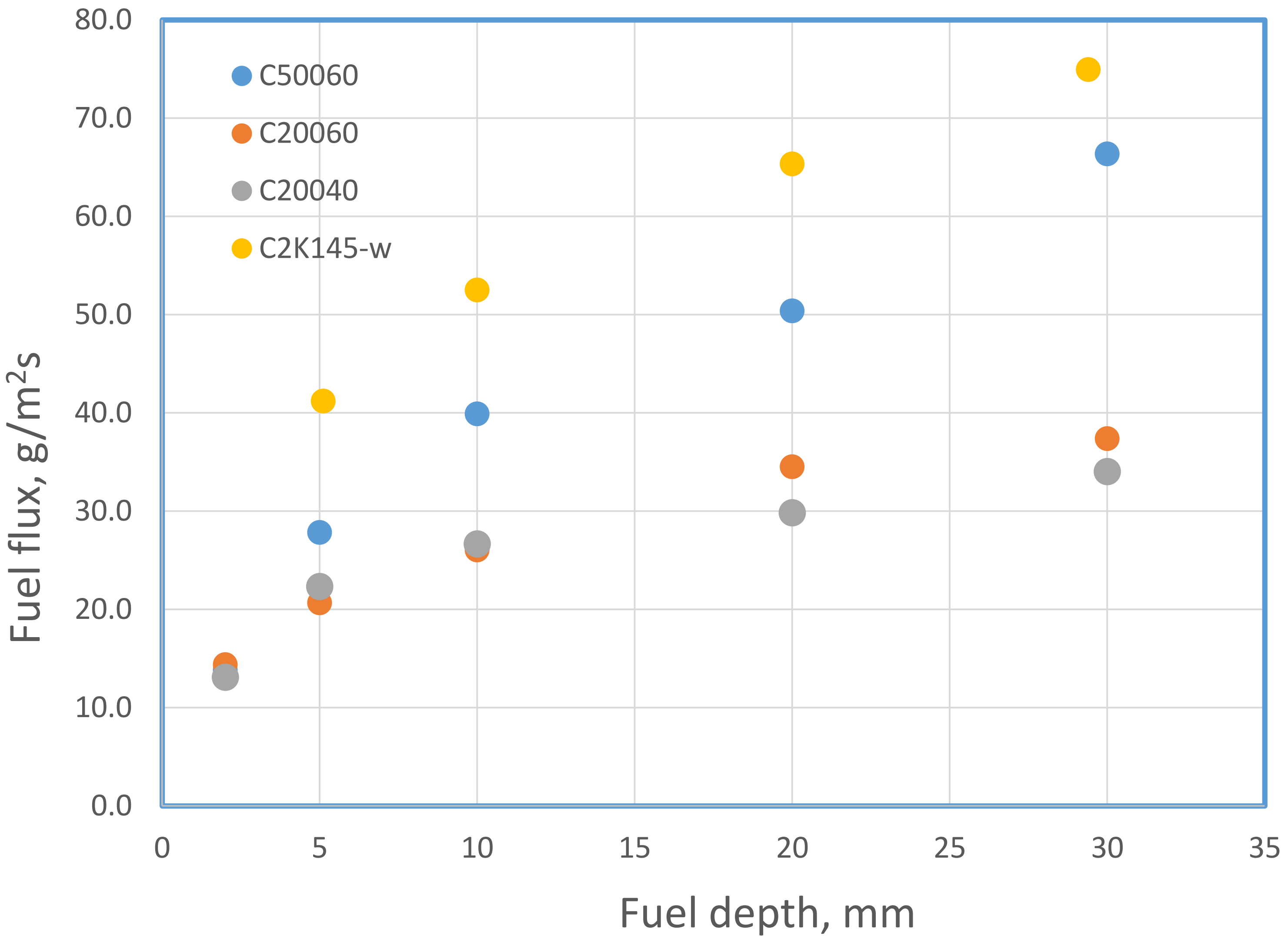
Free board mm	Δfuel flux /Δfreeboard $g/m^2s/mm$
20	0.35
10	0.25
5	0.20
2	0.15

Fuel thickness effect – 500 mm dia pan



The slopes are the mass vs time vary with time – note the complex variation of the burn rates with time
This has been studied by others also.

Fuel flux with fuel depth

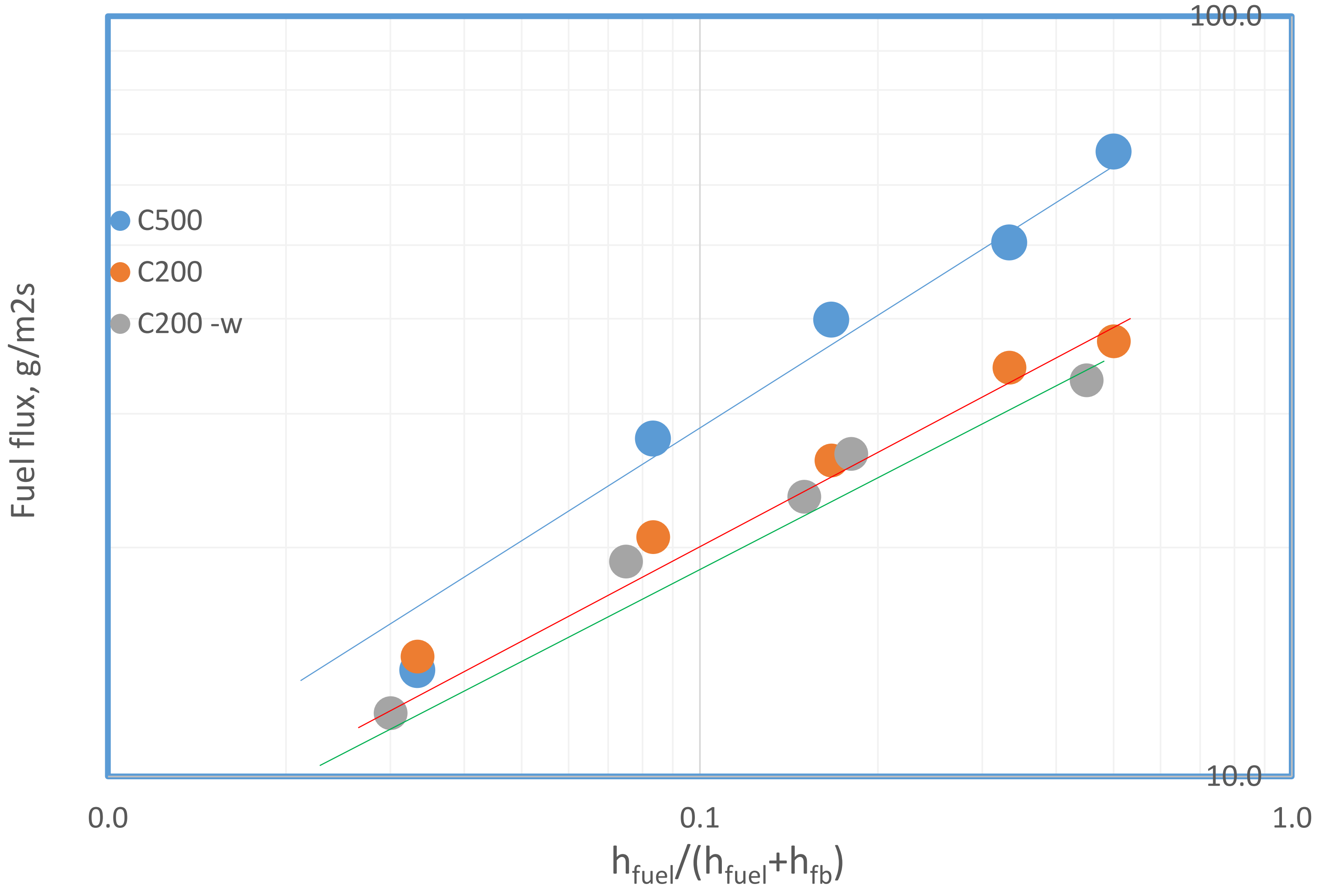


Fuel flux depends strongly on the fuel depth – not recognized explicitly in the literature…

During burn, fuel depth decreases while free board increases. There appears a mutuality.

Will things scale with $h_{fuel}/(h_{fuel}+h_{fb})$? ……

Fuel flux in terms of scaled fuel height



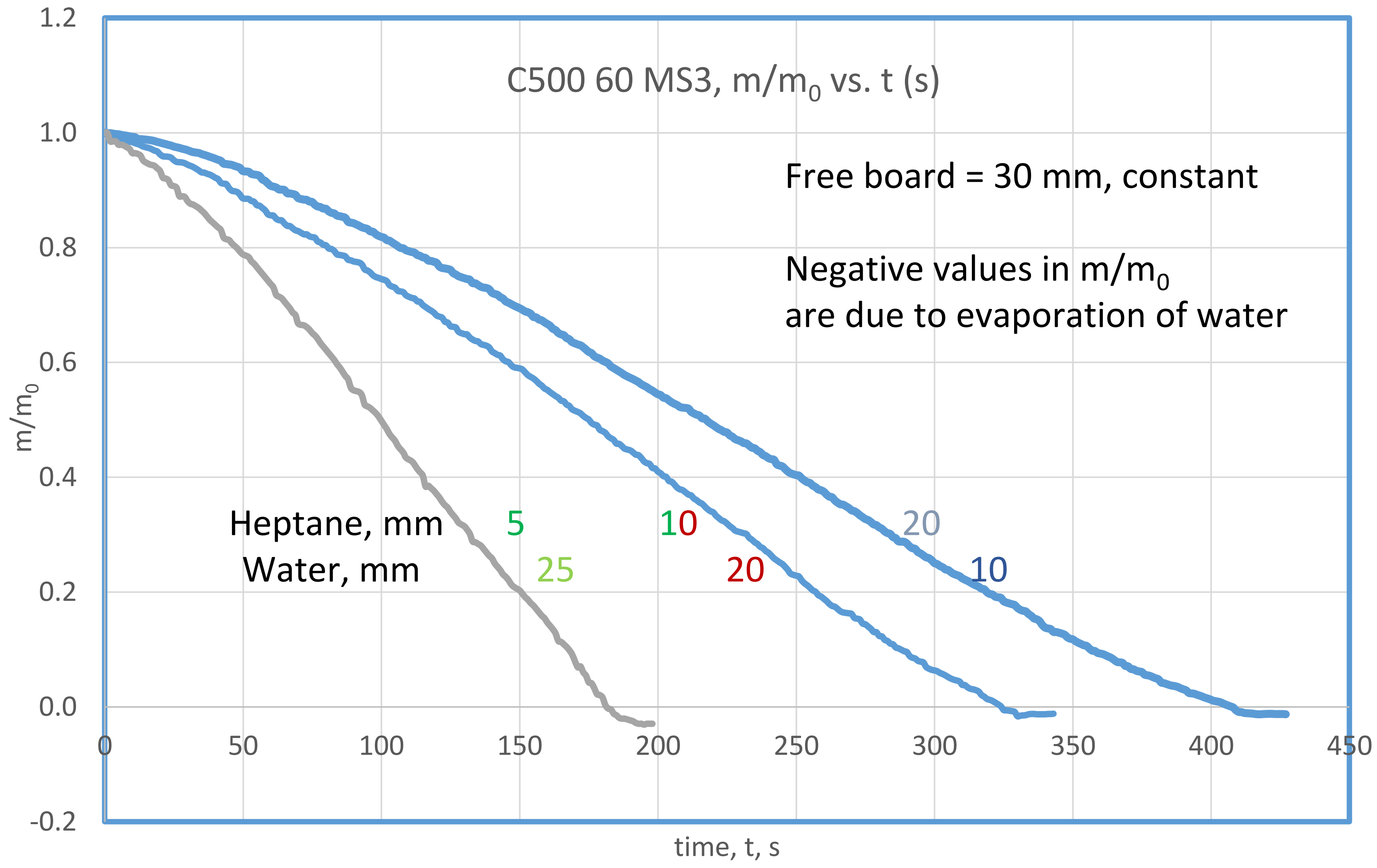
C500 (no water):
mass flux (g/m^2s) = 73 $[h_{fuel}/(h_{fuel}+h_{fb})]^{0.45}$

C200 (no water) :
mass flux (g/m^2s) = 45 $[h_{fuel}/(h_{fuel}+h_{fb})]^{0.36}$

The scaling laws seem reasonable. The exponents seem to depend on the pan size.

It therefore seems scaling of depth should involve the pan size for even as large a size as 2 m where only radiation is supposed to matter!

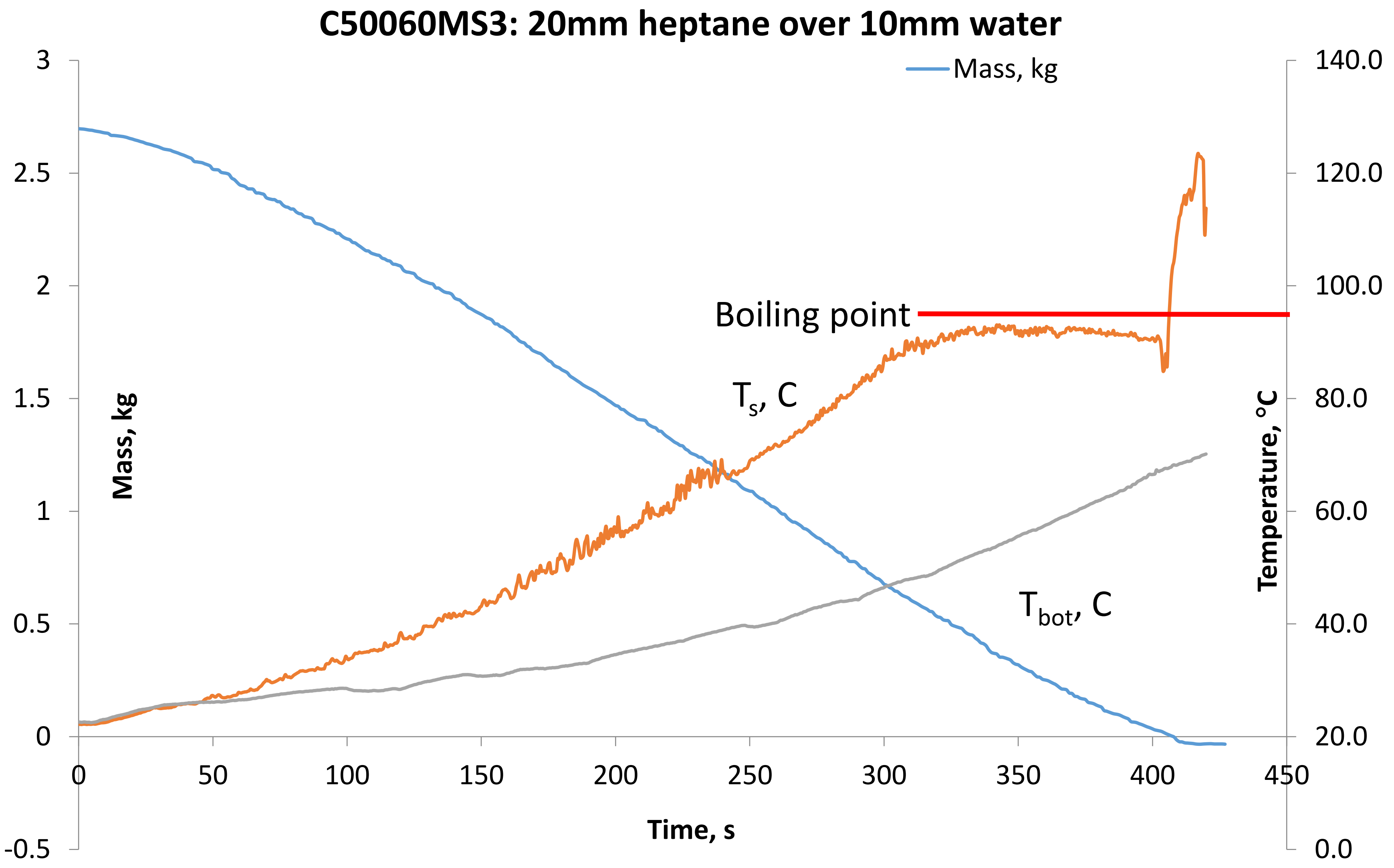
Effect of water depth



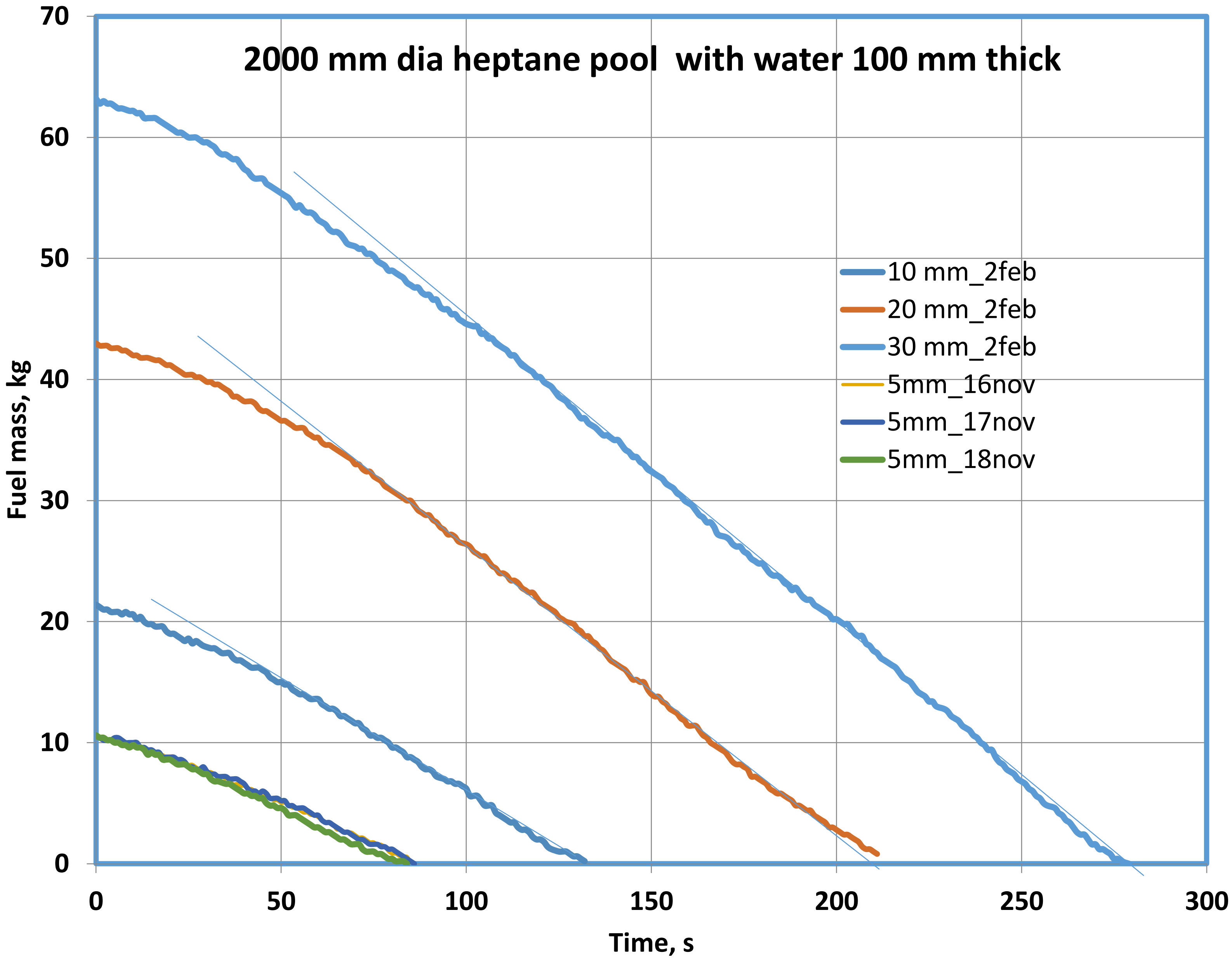
By extracting the effect of free board from earlier data one can deduce the effect of water layer.

For water layer of 2 mm

h_{fuel} mm	Δfuel flux /Δfreeboard g/m^2s per mm
10	1.5
20	1.3
30	0



The surface temperature (T_s) increases from 23 C to 92 C (BP of n-heptane) in 320 s and the bottom wall is also increasing to 65 C. water evaporation is expected beyond 60 C (surface) even if in small amounts


2000 mm dia heptane pool with water 100 mm thick
Fuel mass, kg
Time, s
0
10
20
30
40
50
60
70
0
50
100
150
200
250
300
10 mm_2feb
20 mm_2feb
30 mm_2feb
5mm_16nov
5mm_17nov
5mm_18nov

So,

If even at 2 m size, there are fuel depth effects, there ought to be features related to the liquid on the surface and in depth that must matter.

What are these? There have been studies in this regard by several researchers that show several features

From: Chen, B., Lu, A.X.,Li, C.H., Kang, Q,S., and Lecoustre, V., Initial fuel temperature effects on burning of pool fire, J. Hazardous materials, 188, 369 – 374, 2011 ...200 mm dia pan

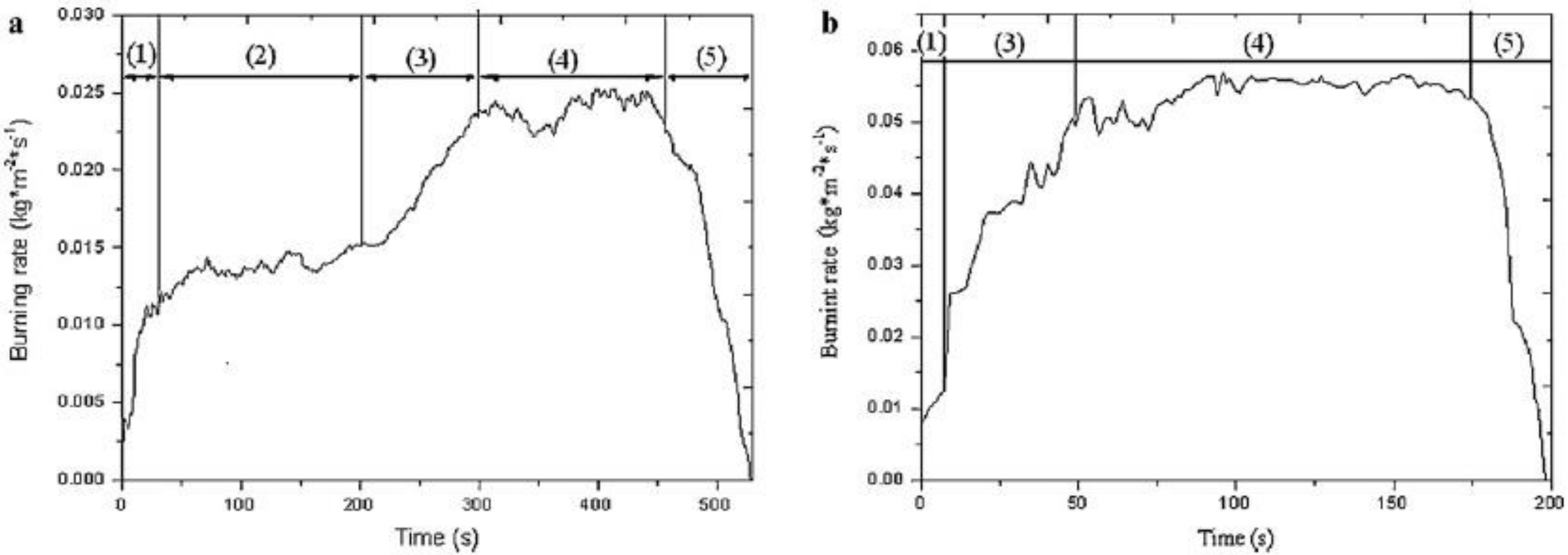


Fig. 4. Temporal evolution of n-heptane pool fire burning rate of same diameter $D = 200$ mm, but different initial temperature: (a) $T_{f,0} = 290$ K; (b) $T_{f,0} = 365$ K.

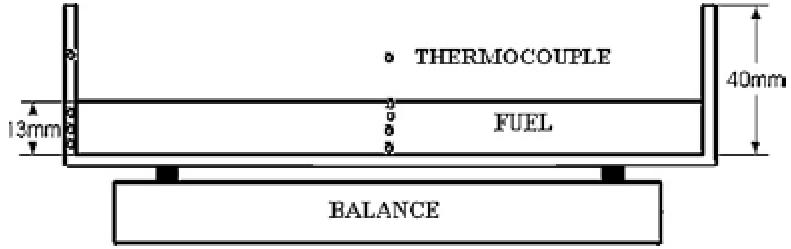


Fig. 1. Schematic diagram of the apparatus.

1. Initial development stage lasting 20 to 30 s
2. Steady burning stage, surface boiling starts to appear
3. Transition stage corresponding to sharp(er) increase in burn rate
4. Bulk boiling stage when bubbles are found every where in the fuel
5. Decay period

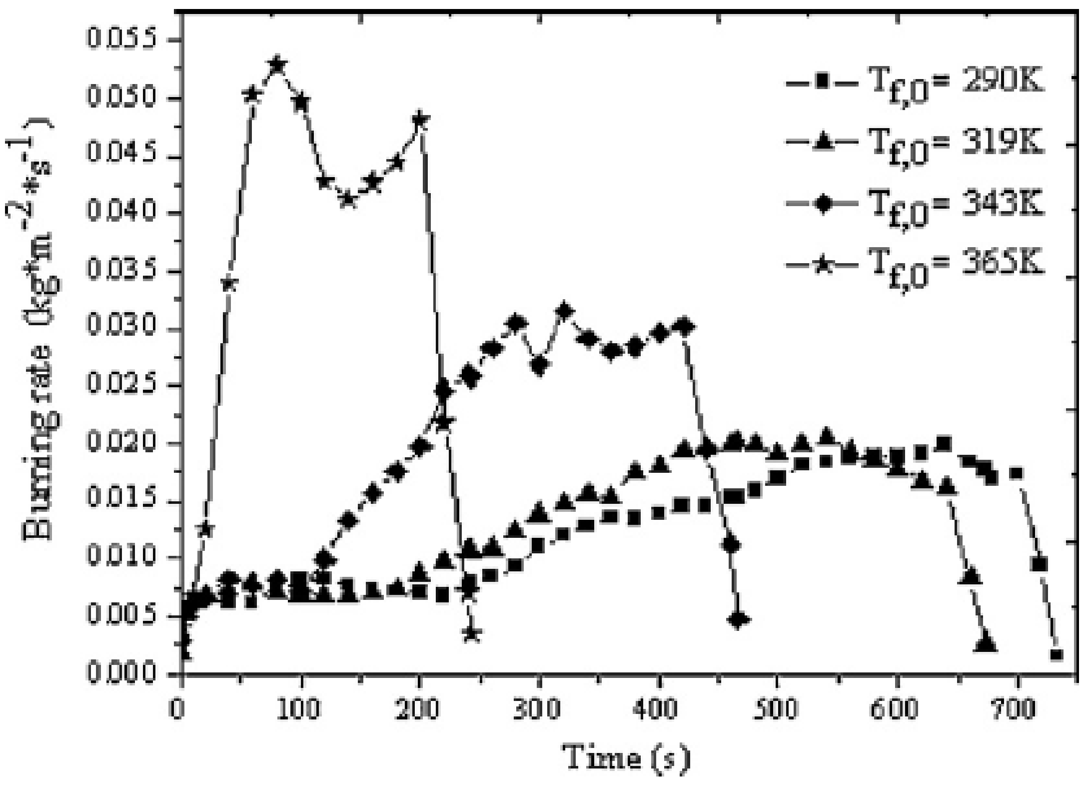


Fig. 5. Burning rates for the 100mm heptanes pool fires.

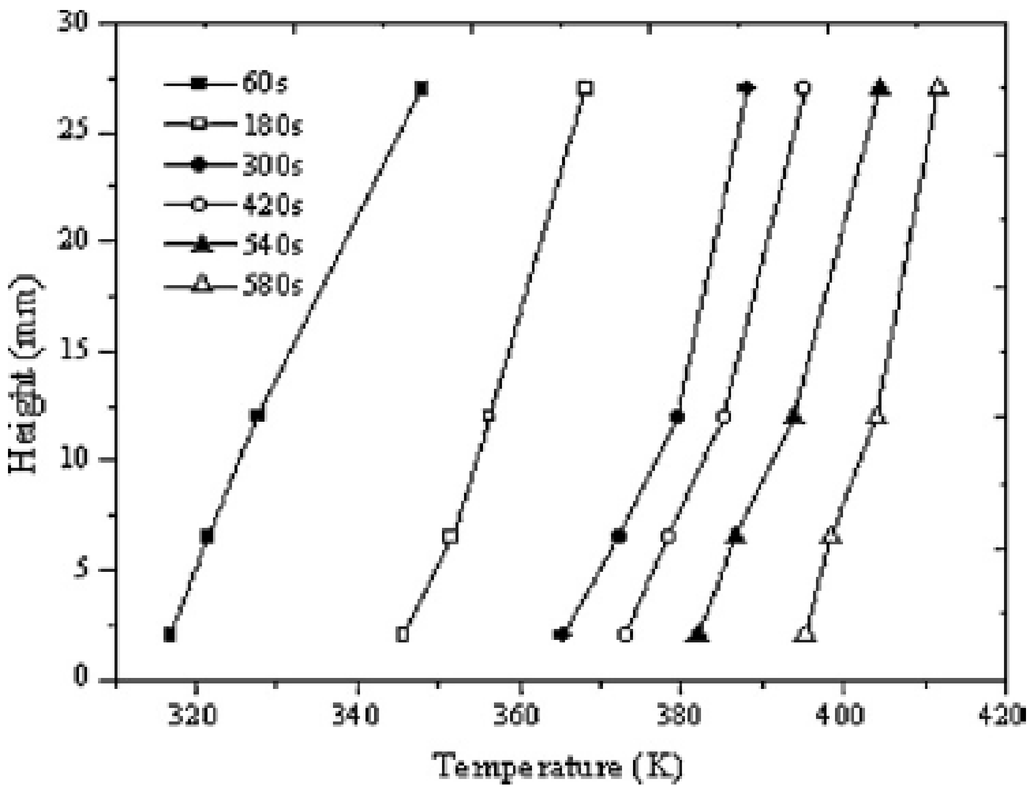


Fig. 8. Vertical temperature distribution of the vessel wall, $D = 200$ mm, $T_{f,0} = 290$ K.

It appears that the entire fuel vaporization process during the fire is transient.

From: Hiroshi Hayasaka: Unsteady burning rates of small pool fires, Fire safety science – Proc. 5th international symposium, pp 499 -510.

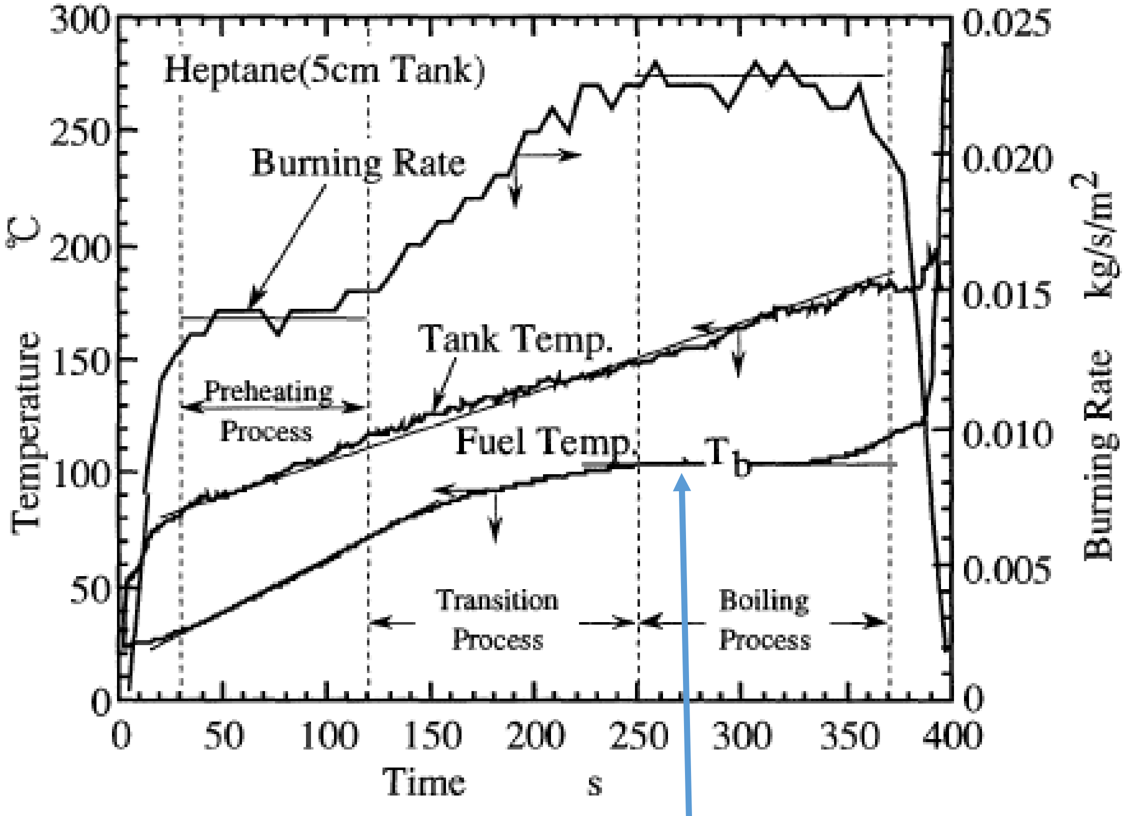
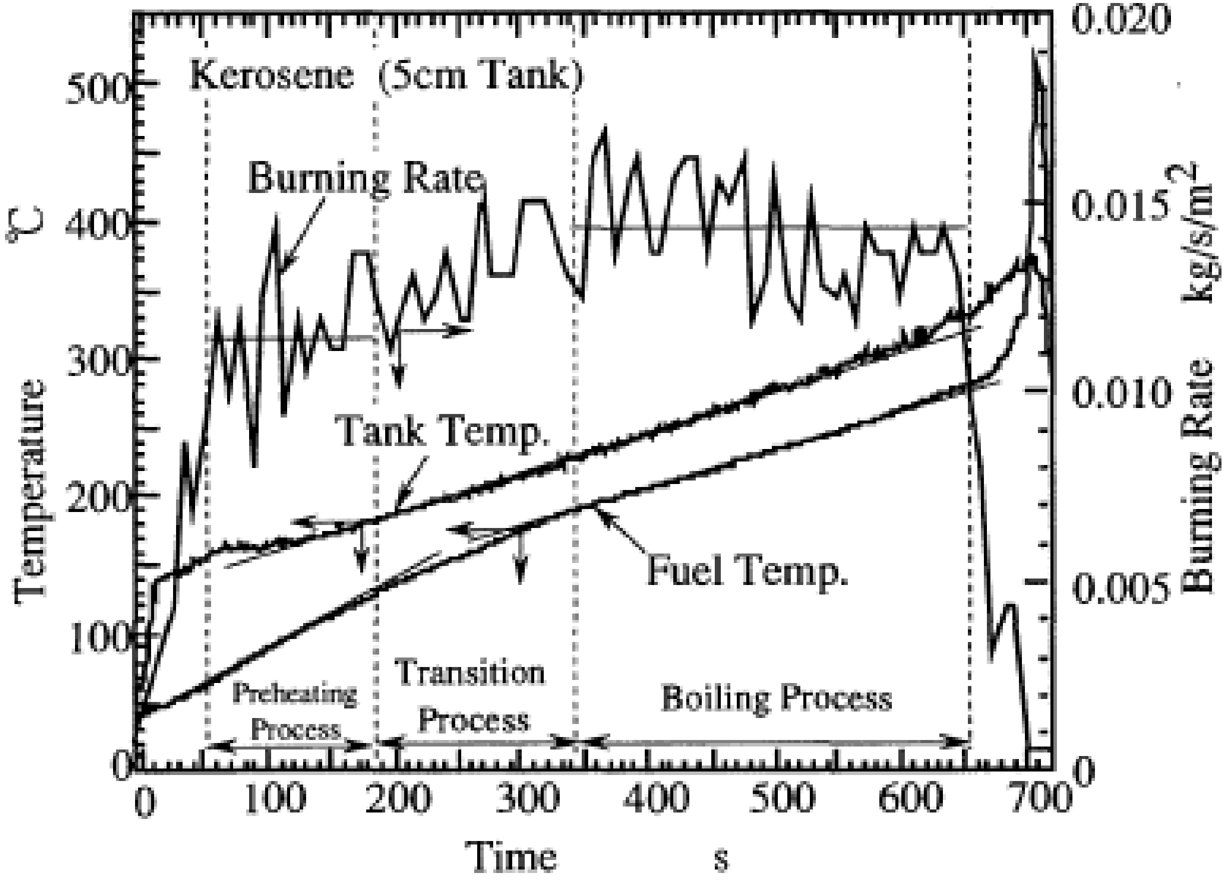


Fig.2 Experimental Results of Heptane



In the case of n-heptane, the fuel reaches the fixed boiling point

In the case of Kerosene, fuel temperature keeps increasing continuously. This is because kerosene is not a pure fuel. Different fragments vaporize at different boiling points

- The thermal profile in the liquid is such that hot region is above the cold region.
- The differences in surface tension due to minor temperature differences caused by differences in heat flux across the surface causes a flow.
- These are called Marangoni flows
- **The scaling parameter is $\alpha_{Tl}\,\mu_l/\sigma_l = (k_l\mu_l/\rho_l c_{pl}\sigma_l)$. It is a scaling length that influences the liquid flow.**

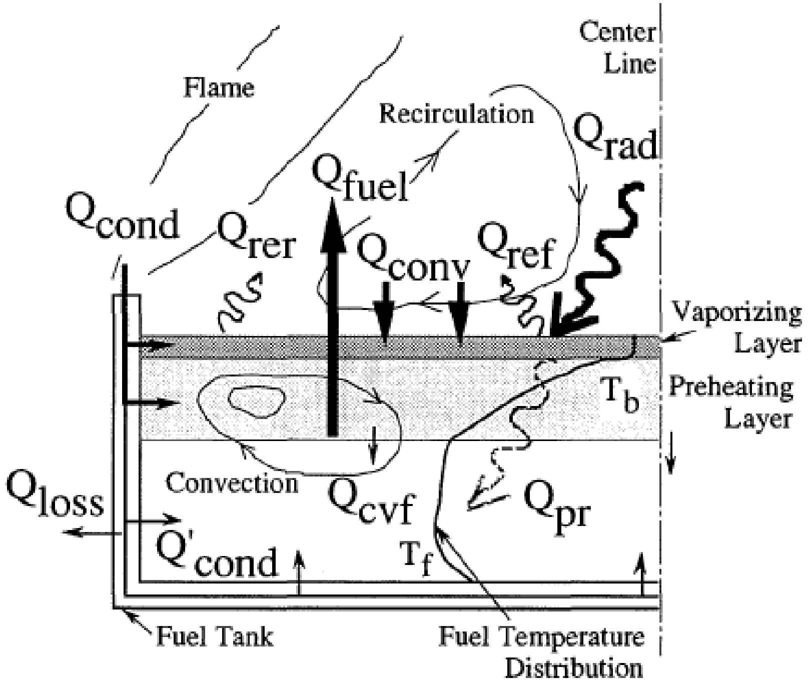
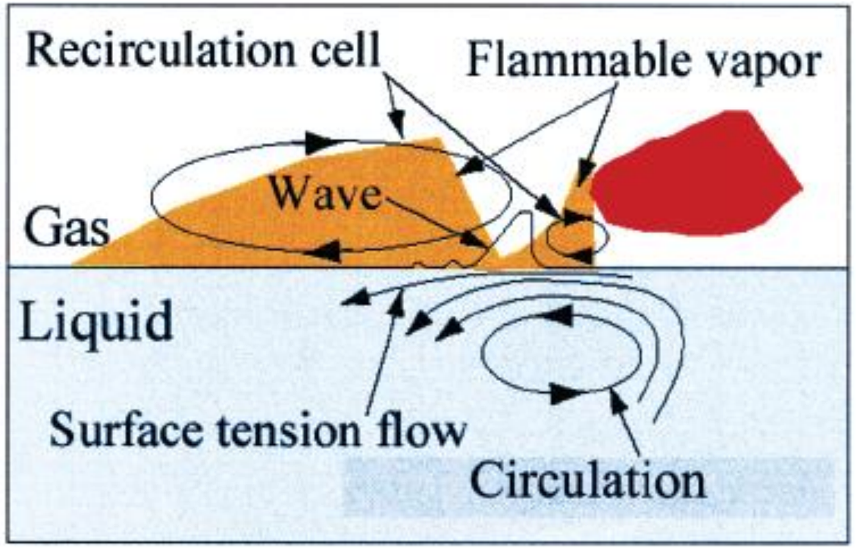
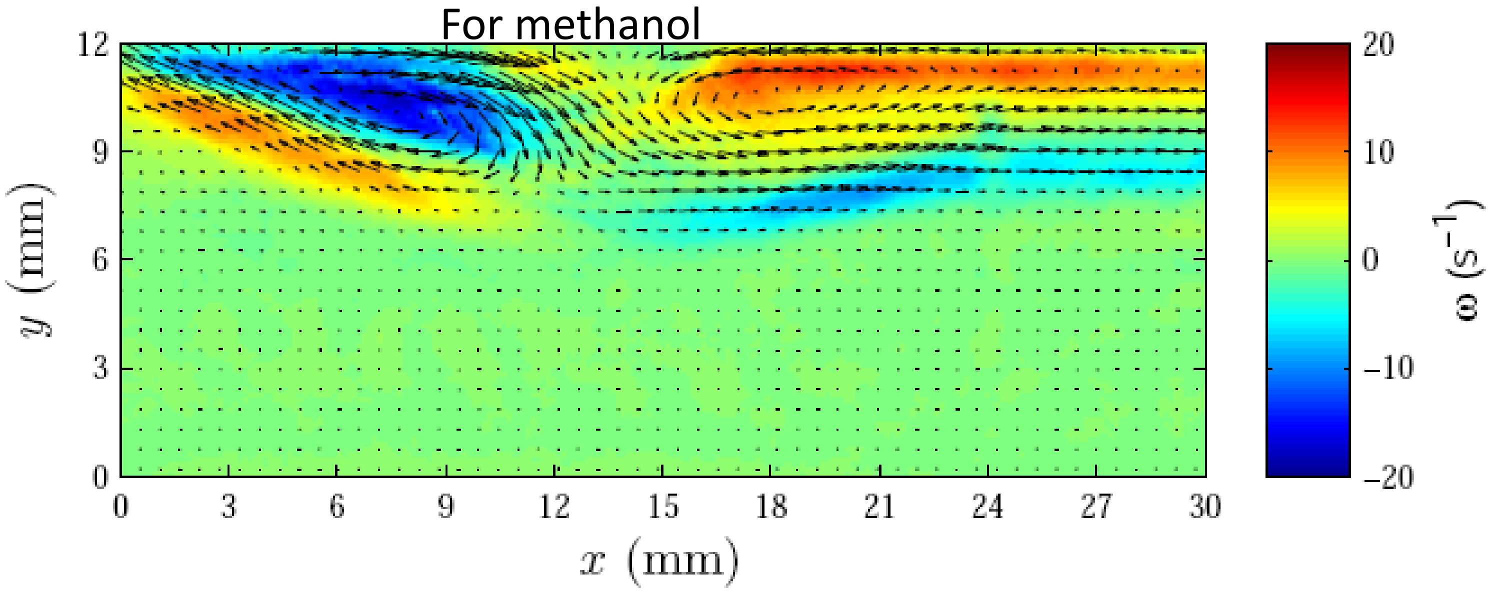
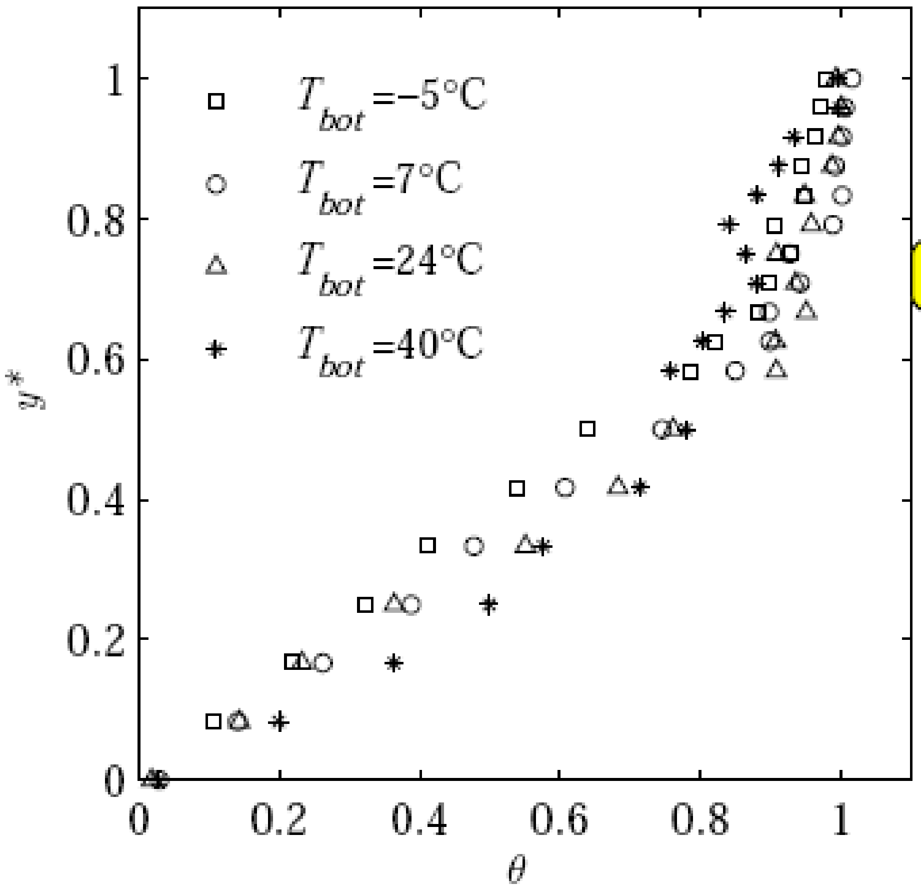


Fig.5 Major Heat Flows near Fuel Surface







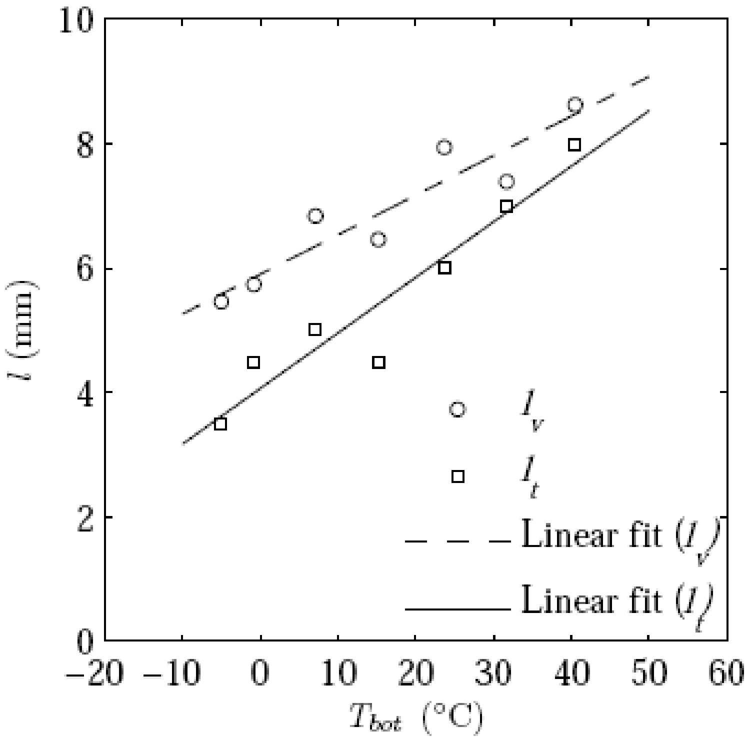


Fig. 6 Thermal and velocity thickness of the top liquid layer of pool fire liquid phase

From Vali, A., Nobes, D. S., and Kostiuk, L. W., Transport phenomena within the liquid phase of a laboratory scale circular methanol pool fire, C & F, 161, pp 1076 – 1084, 2014

The zone near the top layer attains near boiling conditions and the thermal profile propagates into the interior - transient behavior.

Properties of fluids considered –

surface tension- viscosity – thermal diffusivity based length scale and thermal properties of the fluids

Fluid	ρ_l	c_{pl}	L	k_l	σ_l	$\mu_l \times 10^6$	T_b	$c_{pl}(T_b-T_0)/L$	$(k_l\mu_l/\rho_l c_{pl}\sigma_l)$
	kg/m^3	kJ/kg K	kJ/kg	W/mK	N/m	N s/m^2	K		mm
n-heptane	680	2.22	322	0.13	0.02	409	370	0.48	1.76
Diesel	850	1.9	250	0.15	0.028	2400	470	1.29	7.96
CH_3OH	791	2.5	1100	0.21	0.023	593	338	0.09	2.74
Kerosene	820	2	250	0.13	0.028	2400	450	1.2	6.79

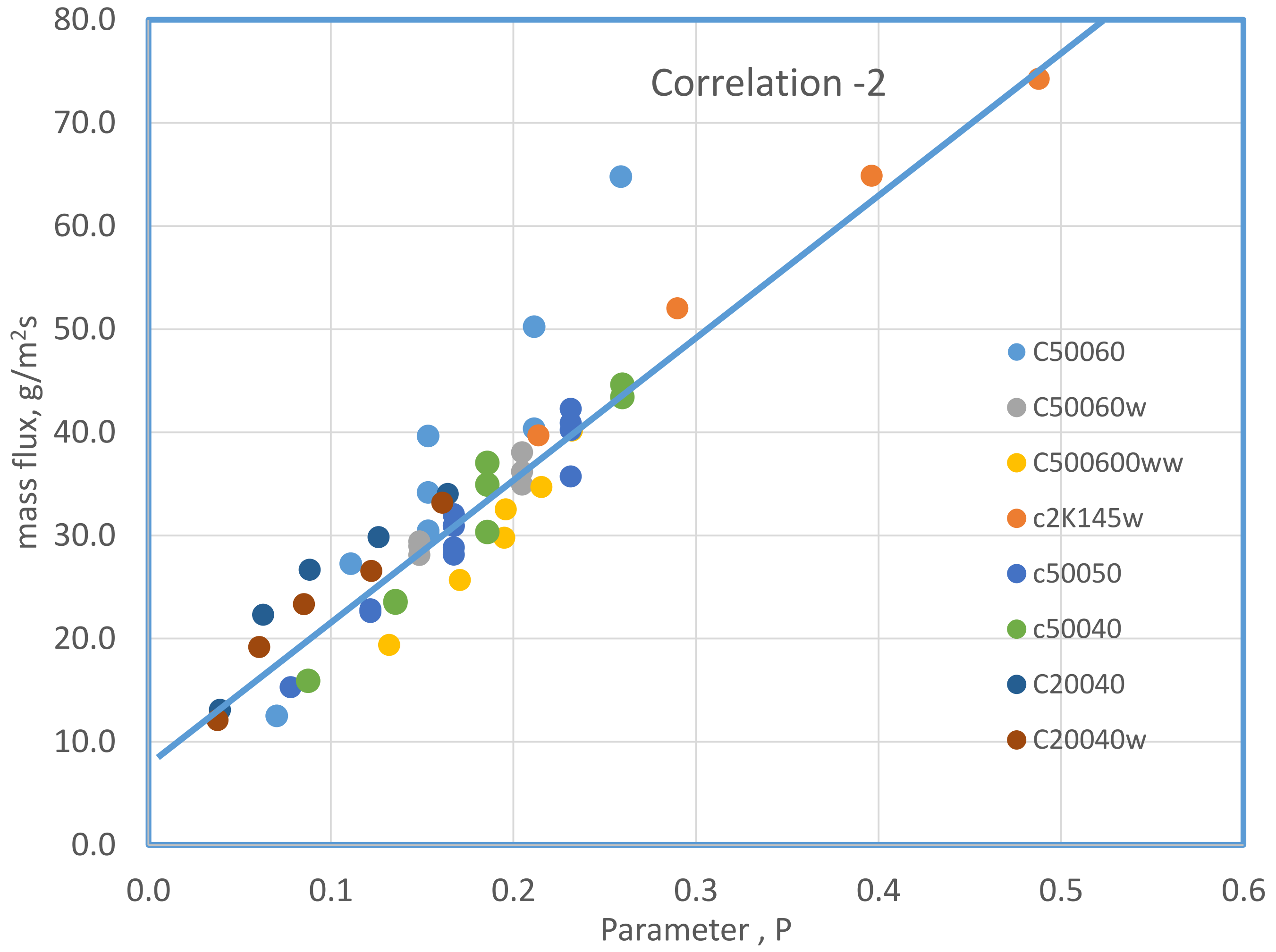
Note that the dimensionless parameters show substantial differences between different fuels; these must be accounted for as well

How do we proceed from here?

There are two pathways

1. Treat the entire process as unsteady – largely arising out of liquid phase. Gas phase adjusts itself to burn rate variations instantaneously. Calculate the g-phase radiational feed back using pulsations – ideas of pink noise (*1/f*). This work is complete (not discussed here). Treat the unsteady conduction through walls, heat transfer to the liquid and water, estimate through Marangoni effects the hot layer propagation in the liquid, unsteady heat transfer to water when used step-by-step in time till burn out occurs. This is accurate as a procedure.
2. Take note that what is intended is an estimate of the burn rate – depth/vaporization time as a function of various parameters – pan diameter, free board, liquid layer, fuel initial temperature and obtain a correlation using scaling principles – some of these have been discussed earlier. By trying out various insights, a correlation has been "developed"

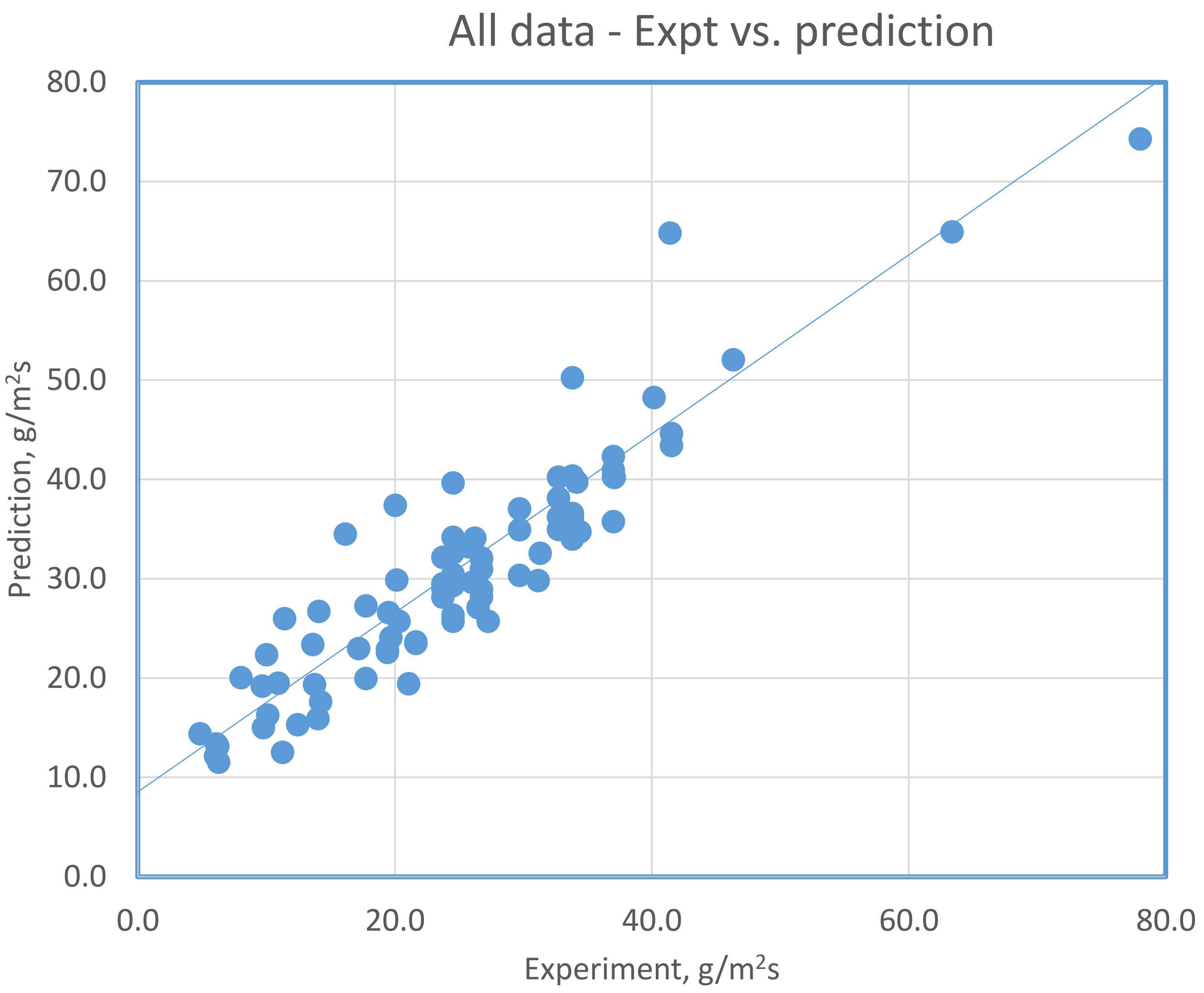
This may not be the most satisfactory one (certainly in my view), but accepted at this time.



This correlation includes classical idea of radiation heat transfer to the fuel surface and other complex effects due to fuel thickness, free board, and water layer.

Other effects included (bur not shown here) are pan wall effect and the nature of fuel.

$$\dot{m}''_{fuel} = 160[1 - e^{(1.6d)}]\frac{\sqrt{h_{fuel}}}{\sqrt{h_{fuel}} + \sqrt{h_{fb}}}\frac{1}{\sqrt{[\alpha_{T,l}\mu_l/\sigma h_{fuel}]}[h_{fb}/d]^{0.75}}\frac{1}{1 + 0.2[h_{water}/d]^{0.25}}$$



…………………Seems satisfactory at present.

What has been done and what may it mean?

- Quantitative and consistent exploration of the various parametric effects on the pool fire burn phenomenon through specifically designed experiments and guidance from scaling principles
- Experiments here as well from elsewhere have shown that the phenomenon is transient with the time scales set by the liquid vaporization process, a feature recognized in many earlier studies but not cogently set out and quantitatively explored.
- The first exploration aimed at obtaining a simple correlation for the mass burn rate as a function of the controlling parameters has been completed.
- It is considered important to construct the unsteady behavior that can capture the observed behavior and will be dealt with in coming times.
- It is not often that such a "green field" problem is encountered and it has surely been an exhilarating experience studying it. It is partly because fire research has not drawn as much attention as classical combustion research where quite often only incremental advances are the primary aim – simpler to do because there are established tracks.
- Here, therefore is an invitation to go where "only not many have gone before"….

……………..Thanks for your attention

8 m JP4 pool fire – Sandia labs, USA

Fuel mass flux = 60 g/m^2s
Power = 124 MWth

Sandia laboratory experiments up to 100 m LNG fires